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Convective amplification of parametrically generated acoustic wave in a piezoelectric semiconductor plasma

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Abstract. Using hydrodynamic model of semiconductor plasmas and coupled-mode theory of interacting waves, we have analytically investigated parametric interaction in a magnetised piezoelectric semiconductor plasma in non-relativistic domain. The temperature dependence of momentum transfer collision frequency of electrons due to their heating by the pump is assumed to induce nonlinearity in the medium. We have derived a dispersion relation which finally gives four unstable acoustic modes; two forward amplifying modes and two backscattered attenuating modes. We have also obtained an expression for the critical pump amplitude ($\mathbf{E}_{\rm cr}$) at and around which gains and phase velocities of amplifying acoustic modes become least dependent on the pump amplitude $\mathbf{E}_0$ and static magnetic field $\mathbf{B}_{\rm s}$. The required $\mathbf{E}_{\rm cr}$ can be readily obtained from a pulsed 10.6 μm CO_2 laser. The magnetic field is found to shift the critical point towards lower pump amplitudes.

PACS. 72.30.+q High-frequency effects; plasma effects – 52.35.Mw Nonlinear phenomena: waves, wave propagation, and other interactions (including parametric effects, mode coupling, ponderomotive effects, etc.) – 72.55.+s Magnetoacoustic effects

1 Introduction

After the advent of laser, parametric interaction (PI) has emerged as one of the most significant sub-field of nonlinear optics. It is an important mechanism of nonlinear mode conversion from electromagnetic to electrostatic and from high to low frequency waves and *vice versa*. It is also one of the key processes to generate coherent radiation at new wavelengths, in the range from extreme ultraviolet to millimeter, not available from direct sources.

Presently, a large number of workers have confined their attention to the parametric processes with Ti:sapphire (Ti:Al_2O_3) which has broadened the range of tunability in the infrared region (0.7–1.06 μm) and thus replacing the widely used conventional dye and colour-centre lasers [1]. Tunable coherent radiation has long been used for biomedical instrumentation, study of energy spectrum of matter and optical data storage. Recently, it has attracted renewed interest in connection with variety of applications in combustion diagnostics, process control, remote sensing and environmental monitoring. Picosecond and femtosecond pulses of tunable radiation can be used in time-resolved studies of chemical reactions and carrier dynamics in semiconductors.

Although PI of waves has been extensively studied in the last four decades, there are tremendous possibilities of further exploration and exploitation. The current trends in the field indicate that this old but fascinating phenomenon is still hotly pursued by both theoreticians and experimentalists and increasing number of interesting applications exploiting PI are being discovered or yet to be discovered [1,2].

The parametric excitation of low frequency waves in semiconductor plasmas has been widely prevailing phenomenon and thus got attention of large number of workers [3–8]. Maheshwari and Tarey [3] first time predicted the resonant excitation of helicon waves in solid state plasma using fluid model. Neogi and Ghosh [4] analysed the problem of parametric amplification of acousto-helicon wave in magnetised piezoelectric semiconductor (PES) by assuming that the origin of the nonlinear interaction lies in second-order optical susceptibility arising due to induced nonlinear current density. Mamun and Salimullah [5] theoretically investigated the parametric excitation of Alfven and helicon waves in semiconductors. Parametric excitation of hypersonic wave in cubic PES with strain-dependent dielectric constant was reported by Artemenko and Sevruk [6]. Anwar *et al.* [7] predicted threshold pump field for the onset of absolute instability of upper-hybrid acoustic wave. Very recently, Sharma and Ghosh [8] have reported an analytical study of frequency modulational interaction between copropagating high frequency pump

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and acoustic wave (AW) and consequent amplification of the modulated waves in a PES plasma. In all the earlier studies, carrier heating by the pump has normally been neglected. However, it is a fact that the propagation of an intense pump through a medium causes heating of the carriers. As a result, these carriers acquire a temperature above that of the lattice. This carrier heating invariably modifies momentum transfer collision frequency (MTCF) of the carriers which is expected to have striking effects on the propagation and amplification characteristics of the interacting modes. The necessary use of high power laser to observe and study higher order nonlinear phenomena has further led to realise that the heating of carriers by such an intense laser are too significant to be ignored, and thus started drawing considerable attention [9,10]. Lee and Cho [9] recently studied the role of thermal coupling of electrostatic waves during PI in the medium. More recently, Wang Jinsong *et al.* [10] investigated the stimulated Brillouin scattering initiated by thermally excited AWs in absorptive media by using distributed fluctuating source model.

Motivated by the renewed interest in the field due to availability of increasingly high power lasers and subsequent necessity to incorporate heating effects in the analysis, in the present paper, we have presented an analytical investigation of convective amplification of parametrically generated AW in a magnetised n-type PES plasma at 77 K. We have employed the well-known hydrodynamic model of one-component semiconductor plasmas [11,15] and coupled-mode theory which is a simple, very instructive and widely used theoretical approach to study PI in semiconductors. There are several excellent reports available in literature based on this model [4,5,8, 12–15]. The heating of carriers by the pump modifies their MTCF and induces nonlinearity in the medium. As far our knowledge goes, no systematic attempt has yet been made towards such an investigation. The basic equations describing the phenomenon are presented in Section 2. This section also deals with complete theoretical formulation of dispersion relation for the generated AW. Finally, an exhaustive numerical analysis performed for a doped n-InSb duly irradiated by a pulsed 10.6 μm CO_2 laser followed by a detailed discussion is given in Section 3.

2 Theoretical formulation

This section deals with the theoretical formulation of the dispersion relation. We have considered the hydrodynamic model ($k_a\ell \ll 1$; k_a being wave number of the AW and ℓ being the electron mean free path) of a homogeneous one-component semiconductor plasma of infinite extent. The medium is considered to be an n-type III-V PES immersed in a static magnetic field $\mathbf{B}_s$ pointing along z-axis. This medium is irradiated by an intense pump expressed using plane wave approximation as

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \exp[\mathrm{i}(k_0 x - \omega_0 t)]. \tag{1}$$

In solids, there are basically two different kinds of plasmas which may be distinguished by their physical properties. One of these is known as immobile plasma; it contains one sign of charge which is fixed to the lattice position, the other charge particle being free to move through out the solid. This plasma is also called one-component plasma because only one of the component is free to move. For example, the electrons in metals, the electrons (or holes) in extrinsic semiconductors or those in a doped semimetals. The other kind of plasma is known as mobile plasma because it possesses carriers of both signs which are free to move in the crystal. In heavily doped n-type III-V compounds, the hole concentration is much smaller than the electron concentration. In addition to this, in these semiconductors the effective mass of holes is usually much larger than that of the electrons. Hence the material can be represented with reasonable accuracy as a single-component free-electron plasma in the back ground of positively charged ions fixed to the lattice points [16]. Here the use of hydrodynamic model enables us to consider the charge carriers in the plasma as a certain continuous medium, a conducting liquid placed in the crystalline lattice. In this model, it is assumed that although the values of the plasma particle velocities can be most diversed, a certain particle velocity distribution is established within the limits of some macroscopic element of the liquid (called a liquid particle), which corresponds to a definite average velocity. In order to obtain the electric currents and the fluxes of matter in the plasma, the equation of motion of liquid particles is used. If there is only one kind of mobile particles in the plasma, the equation of motion of carriers and other dynamical equations in the hydrodynamic approximation governing the PI are

$$\frac{\partial^2 u}{\partial t^2} + \frac{\beta}{\rho}\frac{\partial E_a}{\partial x} = \frac{c}{\rho}\frac{\partial^2 u}{\partial x^2}, \tag{2}$$

$$\frac{\partial \mathbf{v}_0}{\partial t} + \mathbf{v}_0 \cdot \frac{\partial \mathbf{v}_0}{\partial x} + \nu \mathbf{v}_0 = \frac{e}{m}[\mathbf{E} + \mathbf{v}_0 \times \mathbf{B}_s], \tag{3}$$

$$\frac{\partial \mathbf{v}_1}{\partial t} + \mathbf{v}_0 \cdot \frac{\partial \mathbf{v}_1}{\partial x} + \mathbf{v}_1 \cdot \frac{\partial \mathbf{v}_0}{\partial t} + \nu \mathbf{v}_1 = \frac{e}{m}[\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_s]. \tag{4}$$

Physically, the interaction of intense pump with the crystal generates a transverse acoustic mode (Eq. (2)) in the PES. In this equation, $\mathbf{u}(x,t) = \mathbf{u}\exp[\mathrm{i}(k_a x - \omega_a t)]$ denotes displacement of lattice points from their mean positions, β is the piezoelectric constant, ρ is material density, c is the elastic constant and $\mathbf{E}_a$ is the total space-charge field. These acoustic modes in turn produce a localised density perturbations and the consequent refractive index variations and thus presents an acoustic grating for the incident pump. This acoustic grating diffracts the incident pump and generates additional fields in the medium. The diffracted beam contains side-band waves (SBW) $E_1 \exp[\mathrm{i}(k_1 x - \omega_1 t)]$, where $k_1 = k_a \pm k_0$ and $\omega_1 = \omega_a \pm \omega_0$. The + and − signs correspond to anti-stokes and stokes modes, respectively. Here we have neglected higher-order non-resonant terms ($k_1 = k_a \pm pk_0$ and $\omega_1 = \omega_a \pm p\omega_0$, $p \geq 2$) by assuming a long interaction path for the interacting waves [17]. In our model, the approximation $k_a\ell \ll 1$ means that the time in which the AW travels one wavelength, each electron undergoes many collisions. Thus the average thermal velocity of electrons may be neglected safely. The electron is therefore, assumed to possess only zeroth and first order average drift velocities $\mathbf{v}_0$ and $\mathbf{v}_1$ under the influence of $\mathbf{E}_0$ and $\mathbf{B}_\mathrm{s}$ (Eq. (3)) and $\mathbf{E}_1$ and $\mathbf{B}_1$, (Eq. (4)), respectively. Here the parameters ν and m are the modified MTCF of the electrons due to heating by the pump and the electron effective mass, respectively. The external static magnetic field $\mathbf{B}_\mathrm{s}$ is assumed to be so strong that the medium becomes a magnetoplasma ($\omega_\mathrm{p} \sim \omega_\mathrm{c} \sim \omega_0$) and thus allows us to safely neglect the influence of the pump magnetic field.

The electron density perturbations at side-band frequencies couple nonlinearly with the pump to amplify the AW. Under the phase matched conditions ($\mathbf{k}_1 = \mathbf{k}_\mathrm{a} \pm \mathbf{k}_0$ and $\omega_1 = \omega_\mathrm{a} \pm \omega_0$ also known as conservation of momentum and energy, respectively), the AW and SBW both get amplified at the expense of the pump. The electric fields due to charge separation associated with electron density perturbations and the transverse AW give rise to a space-charge field, which may be determined from the respective continuity and Poisson equations:

$$\frac{\partial n_1}{\partial t} + n_0\frac{\partial v_1}{\partial x} + n_1\frac{\partial v_0}{\partial x} + v_0\frac{\partial n_1}{\partial x} = 0, \tag{5}$$

$$\frac{\partial E_\mathrm{a}}{\partial x} + \frac{\beta}{\varepsilon}\frac{\partial^2 u}{\partial x^2} = \frac{n_1 e}{\varepsilon}\cdot \tag{6}$$

Here n_0 and n_1 are the unperturbed and perturbed electron densities, respectively.

2.1 Heating of electrons and modified MTCF

The intense pump when passes through high mobility semiconductor, ions remain passive because of their large inertia while due to low effective masses, electron interact with the pump and gain energy. As a result, electrons attain a temperature somewhat higher than that of the lattice. In steady-state, the electron temperature T_e can be readily estimated from energy balance equation in the following manner.

The power absorbed per electron from the pump is

$$-\frac{e}{2}\mathrm{Re}(\mathbf{v}_0\cdot\mathbf{E}_0^*) = -\frac{e^2\nu}{2m}\frac{(\omega_0^2+\omega_\mathrm{c}^2)}{[(\omega_0^2-\omega_\mathrm{c}^2)^2+4\nu^2\omega_0^2]}E_0E_0^*, \tag{7}$$

where * denotes the complex-conjugate while Re denotes the real part of the quantities concerned. The x-component of $\mathbf{v}_0$ used in the above equation, may be evaluated from equation (3).

The lattice temperature is assumed to be maintained at liquid nitrogen temperature (77 K). Apel *et al.* [18] have concluded after a series of cyclotron resonance experiments that at this temperature phonon scattering dominates over all other scattering mechanisms *viz.* due to ionised impurities, crystal defects etc. in n-InSb. Moreover, in centrosymmetric crystals, the nonpolar phonon scattering is also negligibly small. Thus on account of heavy masses of the acoustic phonons (AP) and relatively larger energies of polar-optical phonons (POP), the dominant scattering mechanisms for the transfers of momentum and energy of carriers may be assumed due to the collisions with AP and POP, respectively [19,20]. Following Conwell [21], the power dissipated per electron in collisions with POP may be expressed as

$$p = \left(\frac{2k_\mathrm{B}\theta_\mathrm{D}}{m\pi}\right)^{1/2} eE_\mathrm{po}x_e^{1/2}K_0(x_e/2) \times \exp(x_e/2)\frac{\exp(x_0-x_e)-1}{(\exp x_0 - 1)}, \tag{8}$$

where $x_{0,e} = (\hbar\omega_\ell/k_\mathrm{B}T_{0,e})$ in which $\hbar\omega_\ell$ is the energy of POP given by $\hbar\omega_\ell = k_\mathrm{B}\theta_\mathrm{D}$; θ_D is the Debye temperature of the medium.

$$E_\mathrm{po} = \frac{me\hbar\omega_\ell}{\hbar^2}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon}\right)$$

is the field of polar-optical scattering potential in which ε and ε_∞ are the static and high frequency permittivities of the medium, respectively. $K_0(x_e/2)$ is the zeroth order Bessel function of the first kind.

In steady-state, the power absorbed per electron from the pump is just equal to the power lost in its collisions with POP. Therefore equations (7, 8) lead us to

$$\left(\frac{T_e - T_0}{T_0}\right) = \frac{e^2\nu}{2m}\frac{(\omega_0^2+\omega_\mathrm{c}^2)\tau}{[(\omega_0^2-\omega_\mathrm{c}^2)^2+4\nu^2\omega_0^2]}E_0E_0^*, \tag{9}$$

where

$$\tau^{-1} = \left(\frac{2k_\mathrm{B}\theta_\mathrm{D}}{m\pi}\right)^{1/2} eE_\mathrm{po}x_0K_0(x_0/2)\frac{x_0^{1/2}\exp(x_0/2)}{\exp(x_0)-1}\cdot \tag{10}$$

Here we have assumed $T_e \approx T_0$ in view of moderate heating of electrons by the pump.

The modified electron MTCF attributed to AP scattering is given by [21]

$$\nu = \nu_0(T_e/T_0)^{1/2}, \tag{11}$$

in which ν_0 is the MTCF of electrons in absence of the pump.

2.2 Dispersion relation

We differentiate equation (5) with respect to time and employ equations (2–6) in collision dominated regime ($\nu \gg \mathbf{k}_0\cdot\mathbf{v}_0, \omega_\mathrm{a}$). An algebraic simplification yields the equation of electron density perturbation wave as

$$\frac{\partial^2 n_1}{\partial t^2} + \nu\frac{\partial n_1}{\partial t} + \overline{\omega}_\mathrm{R}^2 n_1 + \frac{n_0 e\beta k_\mathrm{a}^2}{m\varepsilon}\frac{\nu^2}{(\nu^2+\omega_\mathrm{c}^2)}u = -\mathrm{i}(k_1+k_0)n_1\overline{E}, \tag{12}$$

where

$$\overline{\omega}_\mathrm{R}^2 = \omega_\mathrm{R}^2\frac{\nu^2}{(\nu^2+\omega_\mathrm{c}^2)} \tag{13a}$$

in which

$$\omega_{\mathrm{R}}^2 = \omega_{\mathrm{p}}^2 + k_{\mathrm{a}}^2 \left(\frac{k_{\mathrm{B}} T_e}{m} \right) \quad \text{with} \quad \omega_{\mathrm{p}} = \sqrt{\frac{n_0 e^2}{m\varepsilon}}$$

being the electron plasma frequency and

$$\overline{E} = \frac{e}{m} \left(1 - \frac{\omega_{\mathrm{c}}^2}{\omega_0^2} \right)^{-1} E_0. \tag{13b}$$

The electron density perturbation (n_1) has two components namely fast ($n_{1\mathrm{f}}$) and slow ($n_{1\mathrm{s}}$). The fast component oscillates at the side-band frequencies ($\omega_1 = \omega_{\mathrm{a}} \pm \omega_0$) while the slow one at the acoustic frequency (ω_{a}) and satisfy the relation

$$n_1 = n_{1\mathrm{f}} + n_{1\mathrm{s}}. \tag{14}$$

Using rotating-wave approximation, we may resolve equation (12) into the fast and slow electron density perturbation waves as

$$\frac{\partial^2 n_{1\mathrm{f}}}{\partial t^2} + \nu \frac{\partial n_{1\mathrm{f}}}{\partial t} + \overline{\omega}_{\mathrm{R}}^2 n_{1\mathrm{f}} = -\mathrm{i}(k_1 + k_0) n_{1\mathrm{s}} \overline{E} \tag{15a}$$

$$\nu \frac{\partial n_{1\mathrm{s}}}{\partial t} + \overline{\omega}_{\mathrm{R}}^2 n_{1\mathrm{s}} + \frac{n_0 e \beta k_{\mathrm{a}}^2}{m\varepsilon} \frac{\nu^2}{(\omega_{\mathrm{c}}^2 + \nu^2)} u = -\mathrm{i}(k_1 + k_0) n_{1\mathrm{f}} \overline{E}. \tag{15b}$$

The above equations infer that the fast and slow components couple to each other *via* the pump amplitude $\mathbf{E}_0$ (or $\overline{\mathbf{E}}$). Thus the presence of intense pump is essential for the onset of parametric interaction. The equation (15a) can be simplified to yield

$$\begin{aligned} n_{1\mathrm{f}} = &-\mathrm{i}(k_1 + k_0) n_{1\mathrm{s}} \overline{E} \\ &\times \left[\left\{ \overline{\omega}_{\mathrm{R}}^2 - \mathrm{i}\nu(\omega_{\mathrm{a}} + \omega_0) - (\omega_{\mathrm{a}} + \omega_0)^2 \right\}^{-1} \right. \\ &\left. + \left\{ \overline{\omega}_{\mathrm{R}}^2 - \mathrm{i}\nu(\omega_{\mathrm{a}} - \omega_0) - (\omega_{\mathrm{a}} - \omega_0)^2 \right\}^{-1} \right]. \end{aligned} \tag{16}$$

After some algebraic simplification, the above equation reduces to

$$n_{1\mathrm{f}} = \frac{2\mathrm{i}(k_1 + k_0) n_{1\mathrm{s}} \overline{E}}{(\omega_0^2 + \nu^2)}, \tag{17}$$

here we have assumed that $\omega_{\mathrm{a}} \ll \omega_0$ and $\overline{\omega}_{\mathrm{R}} \sim \nu$.

The displacement of the lattice points due to propagation of the AW may be obtained from equation (2) as

$$u = \frac{(\beta e / \rho\varepsilon)}{\left(\omega_{\mathrm{a}}^2 - \frac{c}{\rho} k_{\mathrm{a}}^2 - \frac{\beta^2 k_{\mathrm{a}}^2}{\rho\varepsilon} \right)} n_{1\mathrm{s}}. \tag{18}$$

Using equations (17, 18) in equation (15b), one may obtain the dispersion relation for the AW in the medium as

$$\left(\omega_{\mathrm{a}}^2 - \frac{c}{\rho} k_{\mathrm{a}}^2 - \frac{\beta^2 k_{\mathrm{a}}^2}{\rho\varepsilon} \right) \left[\overline{\omega}_{\mathrm{R}}^2 - \mathrm{i}\nu\omega_{\mathrm{a}} - \frac{2(k_1 + k_0)^2 \overline{E}^2}{(\omega_0^2 + \nu^2)} \right] = - \overline{\omega}_{\mathrm{R}}^2 \frac{\beta e}{\rho\varepsilon} n_{1\mathrm{s}}. \tag{19}$$

This equation represents the general dispersion relation of the AW in PES in the hydrodynamic limit $k_{\mathrm{a}}\ell \ll 1$. Equation (19) also indicates that the piezoelectricity of the medium acts as a coupling parameter between the generated AW and the electrostatic SBW in the medium.

2.3 Convective amplification of the AW and critical pump amplitude $\mathbf{E_{cr}}$

In order to explore the possibility of convective amplification of the AW, we rewrite equation (19) as

$$\left(\omega_{\mathrm{a}}^2 - \frac{c}{\rho} k_{\mathrm{a}}^2 - A \right) + \overline{\omega}_{\mathrm{R}}^2 \frac{A}{G} \left(1 + \mathrm{i}\nu \frac{\omega_{\mathrm{a}}}{G} \right) = 0, \tag{20}$$

where $A = \kappa^2 k_{\mathrm{a}}^2 v_{\mathrm{a}}^2$ in which $\kappa = \beta / \sqrt{c\varepsilon}$ and $v_{\mathrm{a}} = \sqrt{c/\rho}$ being the AW velocity and

$$G = \overline{\omega}_{\mathrm{R}}^2 - \frac{2(k_1 + k_0)^2 \overline{E}^2}{(\omega_0^2 + \nu^2)} \cdot \tag{21}$$

By assuming $|\mathbf{k}_0| \approx |\mathbf{k}_1| \approx |\mathbf{k}_{\mathrm{a}}| = k$, $\omega_{\mathrm{a}} = \omega$, and $G \sim \overline{\omega}_{\mathrm{R}}^2$, equation (20) simplifies to

$$B_1 k^4 - B_2 k^2 + B_3 = 0, \tag{22}$$

where

$$B_1 = \frac{8\kappa^2 v_{\mathrm{a}}^2 \overline{E}^2}{(\omega_0^2 + \nu^2)}, \tag{23a}$$

$$B_2 = (\overline{\omega}_{\mathrm{R}}^2 - \mathrm{i}\nu\omega\kappa^2) v_{\mathrm{a}}^2, \tag{23b}$$

$$B_3 = \omega^2 \overline{\omega}_{\mathrm{R}}^2. \tag{23c}$$

Obviously equation (22) possesses four roots which can predict amplification/attenuation of the generated AWs. In order to solve equation (22), we proceed by treating it as a quadratic in k^2 whose roots are given by

$$k_1^2 = \frac{-\mathrm{i}\nu\omega(\omega_0^2 + \nu^2)}{16\overline{E}^2} + \left[\frac{\overline{\omega}_{\mathrm{R}}^2 (\omega_0^2 + \nu^2)}{8\kappa^2 \overline{E}^2} - \frac{\omega^2}{v_{\mathrm{a}}^2} \right], \tag{24a}$$

$$k_2^2 = \frac{-\mathrm{i}\nu\omega(\omega_0^2 + \nu^2)}{16\overline{E}^2} + \frac{\omega^2}{v_{\mathrm{a}}^2} \cdot \tag{24b}$$

As evident from equation (24a) that the real part of k_1^2 vanishes when

$$\frac{\overline{\omega}_{\mathrm{R}}^2 (\omega_0^2 + \nu^2)}{8\kappa^2 \overline{E}^2} = \frac{\omega^2}{v_{\mathrm{a}}^2} \cdot \tag{25}$$

If we address the pump amplitude satisfying the above condition as critical pump amplitude E_{cr} then

$$\begin{aligned} E_{\mathrm{cr}} &= \frac{m}{e} \frac{\overline{\omega}_{\mathrm{R}} v_{\mathrm{s}}}{2\kappa\omega} \left(1 - \frac{\omega_{\mathrm{c}}^2}{\omega_0^2} \right) \sqrt{\frac{\omega_0^2 + \nu^2}{2}} \\ &= \frac{m v_{\mathrm{s}}}{2\sqrt{2} e \kappa} \frac{\omega_{\mathrm{R}} \nu}{\omega} \left(1 - \frac{\omega_{\mathrm{c}}^2}{\omega_0^2} \right) \sqrt{\frac{\omega_0^2 + \nu^2}{\omega_{\mathrm{c}}^2 + \nu^2}} \cdot \end{aligned} \tag{26}$$

It may be inferred from this equation that with increasing ω_{c}, E_{cr} increases parabolically due to the factor $(1 - \omega_{\mathrm{c}}^2/\omega_0^2)$ but the shape gets slightly modified due to the presence of the term $(\omega_{\mathrm{c}}^2 + \nu^2)^{-0.5}$ in equation (26). Now we seek solutions of equations (24) in different regions of the pump amplitude E_0.

2.3.1 At low pump amplitude [$E_0 < E_{cr}$]

In this region

$$\frac{\overline{\omega}_R^2(\omega_0^2+\nu^2)}{8\kappa^2\overline{E}^2} > \frac{\omega^2}{v_a^2}$$

using De-Moivre theorem, one gets the roots of equations (24) as

$$k_{1f,b} = \pm\left[\frac{\overline{\omega}_R}{2\kappa\overline{E}}\sqrt{\frac{\omega_0^2+\nu^2}{2}} - \frac{i\nu\omega\kappa}{8\overline{\omega}_R\overline{E}}\sqrt{\frac{\omega_0^2+\nu^2}{2}}\right], \quad (27a)$$

$$k_{2f,b} = \pm\left[\frac{1}{4\overline{E}}\sqrt{\frac{\nu\omega(\omega_0^2+\nu^2)}{2}} - \frac{i}{4\overline{E}}\sqrt{\frac{\nu\omega(\omega_0^2+\nu^2)}{2}}\right]. \quad (27b)$$

The subscript f and b to the quantities concerned stand for forward and backward acoustic modes corresponding to + and − sign, respectively. Clearly, the roots (Eqs. (27)) are complex whose real and imaginary parts help in study of propagation characteristics and amplification/attenuation of the AW in the medium, respectively. Clearly, there are four acoustic modes corresponding to these four roots (Eqs. (27)); two propagating in forward direction and remaining two in the backward direction. It may also be inferred that the forward modes amplify (due $k_i < 0$) and the back scattered modes attenuate (due $k_i > 0$) while propagating through the medium.

The phase velocities of these modes are obtained as

$$v_{p1f,b} = \left|\frac{\omega}{(k_{1f,b})_r}\right| = 2\kappa\overline{E}\frac{\omega}{\overline{\omega}_R}\sqrt{\frac{2}{(\omega_0^2+\nu^2)}}, \quad (28a)$$

$$v_{p2f,b} = \left|\frac{\omega}{(k_{2f,b})_r}\right| = 4\overline{E}\sqrt{\frac{2\omega}{\nu(\omega_0^2+\nu^2)}}\cdot \quad (28b)$$

where the subscript r to the quantities represent their real parts.

2.3.2 At critical pump amplitude [$E_0 = E_{cr}$]

At critical pump amplitude, the roots of equation (23a) are

$$k_{1f,b} = \pm\left[\frac{1}{4\overline{E}}\sqrt{\frac{\nu\omega(\omega_0^2+\nu^2)}{2}} - \frac{i}{4\overline{E}}\sqrt{\frac{\nu\omega(\omega_0^2+\nu^2)}{2}}\right], \quad (29)$$

and the phase velocities of the corresponding acoustic modes are

$$v_{p1f,b} = \left|\frac{\omega}{(k_{1f,b})_r}\right| = 4\overline{E}\sqrt{\frac{2\omega}{\nu(\omega_0^2+\nu^2)}}\cdot \quad (30)$$

From equation (24b), one may readily conclude that there is no critical pump amplitude for acoustic modes corresponding to k_2^2. A careful observation of equations (27–30) reveals that at $E_0 = E_{cr}$, a singularity occurs for gains and phase velocities of the AWs corresponding to k_1^2 and k_2^2.

2.3.3 Just above critical pump amplitude [$E_0 > E_{cr}$]

In this region

$$\frac{\overline{\omega}_R^2(\omega_0^2+\nu^2)}{8\kappa^2\overline{E}^2} > \frac{\omega^2}{v_a^2}$$

the roots of equations (23) are obtained as

$$k_{1f,b} = \pm\left[\frac{\nu v_a(\omega_0^2+\nu^2)}{32\overline{E}^2} - i\sqrt{\frac{1}{2}\left(\frac{\omega^2}{v_a^2} - \frac{\overline{\omega}_R^2(\omega_0^2+\nu^2)}{8\kappa^2\overline{E}^2}\right)}\right], \quad (31a)$$

$$k_{2f,b} = \pm\left[\frac{\omega}{v_a} - i\frac{\nu v_a(\omega_0^2+\nu^2)}{32\overline{E}^2}\right], \quad (31b)$$

and the corresponding phase velocities of these acoustic modes are

$$v_{p1f,b} = \left|\frac{\omega}{(k_{1f,b})_r}\right| = \frac{32\omega}{\nu(\omega_0^2+\nu^2)v_a}\overline{E}^2, \quad (32a)$$

$$v_{p2f,b} = \left|\frac{\omega}{(k_{2f,b})_r}\right| = v_a. \quad (32b)$$

3 Results and discussions

To have numerical appreciation of the results obtained in the above analysis, an n-InSb crystal is assumed to be irradiated by 10.6 μm CO_2 pulsed laser. The other parameters used in the numerical estimation are; $m = 0.014m_0$, m_0 is the free electron mass, $n_0 = 10^{22}$ m^{-3}, $\varepsilon_L = 17.54$, $\varepsilon_\infty = 15.7$, $\rho = 5.8\times10^3$ kg m^{-3}, $\beta = 0.054$ C m^{-2}, $T_0 = 77$ K, $\theta_D = 278$ K, $\omega_0 = 1.78\times10^{14}$ s^{-1}, $\nu_0 = 3.5\times10^{11}$ s^{-1}, $\omega = 4\times10^9$ s^{-1}, $k = 5\times10^6$ m^{-1}.

Figure 1 depicts the variations of electron to lattice temperature ratio T_e/T_0 and electron MTCF ν with respect to the pump amplitude E_0. Both the parameters are almost constant up to $E_0 \approx 10^6$ V m^{-1}, but increase sharply afterwards. The shapes of both the curves are parabola which agrees with equations (9, 11), respectively. This infers that at low pump intensities, one may safely ignore heating of the carriers by the pump and may assume $\nu = \nu_0$. However, this is not the case while dealing with high intensity lasers and hence the heating of the carriers by the pump must be given due considerations.

As clear from the analysis in the preceding section that there are four convectively unstable acoustic modes; two forward modes which amplify and the two backward modes that attenuate. Figures 2 and 3 display the variations of gains ($|k_{1fi}|$, $|k_{2fi}|$) and phase velocities (v_{p1f}, v_{p2f}) of the amplifying acoustic modes with respect to the pump amplitude E_0, when $\omega_c = 0.5\omega_0$. The gains $|k_{1fi}|$ and $|k_{2fi}|$ decrease with increasing E_0 and meet at certain critical value of E_0 ($\approx 9\times10^6$ V m^{-1} in the present case). This singularity, corresponding to critical pump amplitude, is expected in Section 2.3.2. In the region $E_0 > E_{cr}$, $|k_{1fi}|$ starts increasing gradually but soon abruptly as shown in Figure 2. The other gain $|k_{2fi}|$ decreases for all values

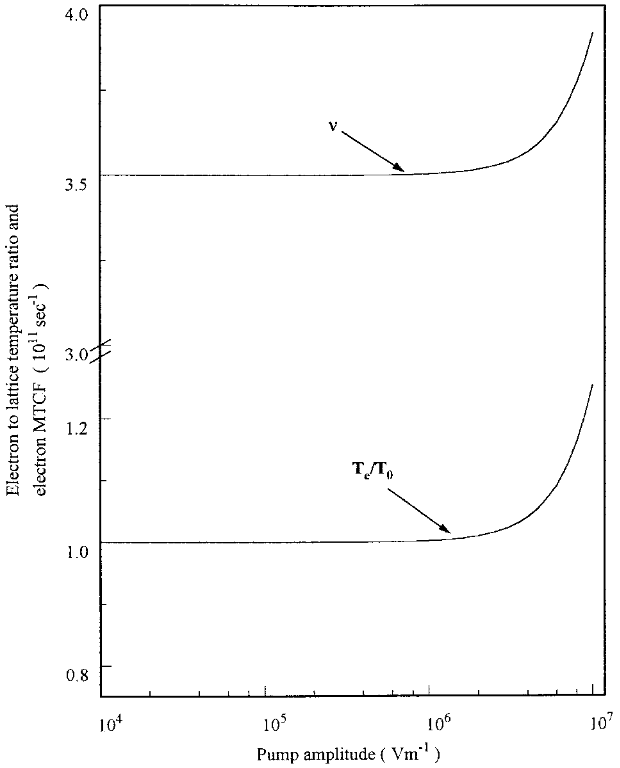

Fig. 1. Variation of electron to lattice temperature ratio T_e/T_0 and momentum transfer collision frequency ν with pump amplitude E_0 when $\omega_c = 0.5\omega_0$ and $n_0 = 10^{22}$ m^{-3}.

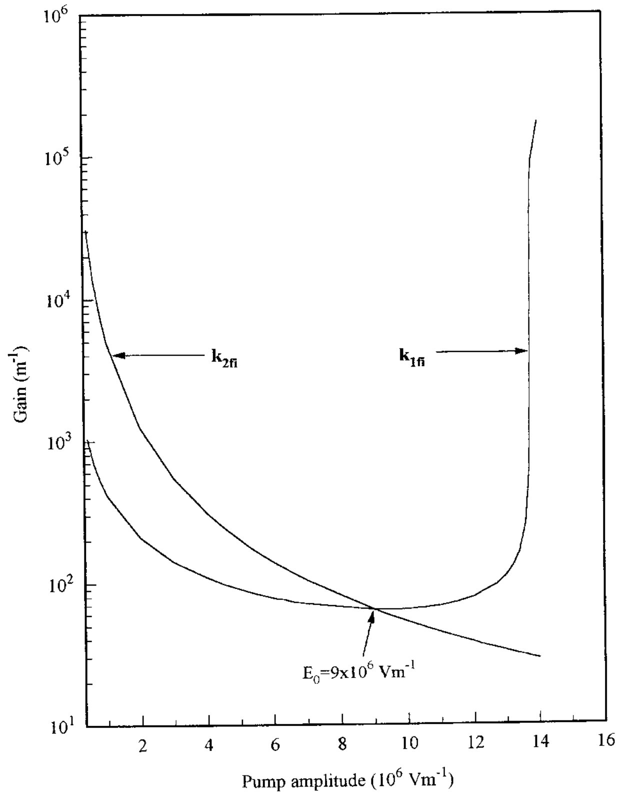

Fig. 2. Variation of gains ($|k_{1\text{fi}}|$, $|k_{2\text{fi}}|$) with pump amplitude E_0 when $\omega_c = 0.5\omega_0$ and $n_0 = 10^{22}$ m^{-3}.

of E_0. The shape of the gain-curves can be conveniently explained from their expressions (Eqs. (27a, 29, 31a) for $|k_{1\text{fi}}|$ and Eqs. (27b, 31b) for $|k_{2\text{fi}}|$) in terms of variations of $\overline{\omega}_{\text{R}}$, $\overline{E}$ and ν in different regions of E_0. In the low pump intensity region (up to $E_0 \approx 3 \times 10^6$ V m^{-1}) the heating of carriers is negligible so ν and $\overline{\omega}_{\text{R}}$ remain almost constant but $\overline{E}$ increases and consequently $|k_{1\text{fi}}|$ and $|k_{2\text{fi}}|$ decrease. As we approach E_{cr}, ν starts increasing considerably and at $E_0 = E_{\text{cr}}$, the variations of $\overline{E}$ and ν nullify each other which results in a plateau as shown in Figure 2. In the region $E_0 > E_{\text{cr}}$, $|k_{1\text{fi}}|$ shoots up due to the factor $1/\overline{E}_0^2$ (Eq. (31a)) while $|k_{2\text{fi}}|$ decreases due to the factor $\nu/\overline{E}_0^2$ (Eq. (31b)). The phase velocity v_{p1f} shows some peculiar response to the variation of E_0 in the sense that although it increases for all allowed values of E_0, the slope of the curve varies continuously. Initially, the slope of the curve decreases until E_{cr} is reached. Beyond this, the slope starts increasing and finally becomes almost upwards as shown in Figure 3. The phase velocity v_{p2f} of the other acoustic mode remains practically constant for all values of the pump amplitude. Again a singularity occurs at $E_0 = E_{\text{cr}}$. It is worthwhile to mention here that Figure 2 might mislead to conclude that the gain $|k_{1\text{fi}}|$ may be increased to any arbitrarily large value by simply increasing E_0 beyond E_{cr}. Unfortunately this never happens because at this pump amplitude the phase velocity $v_{1\text{pf}}$ also increases to such an extent that relativistic effects

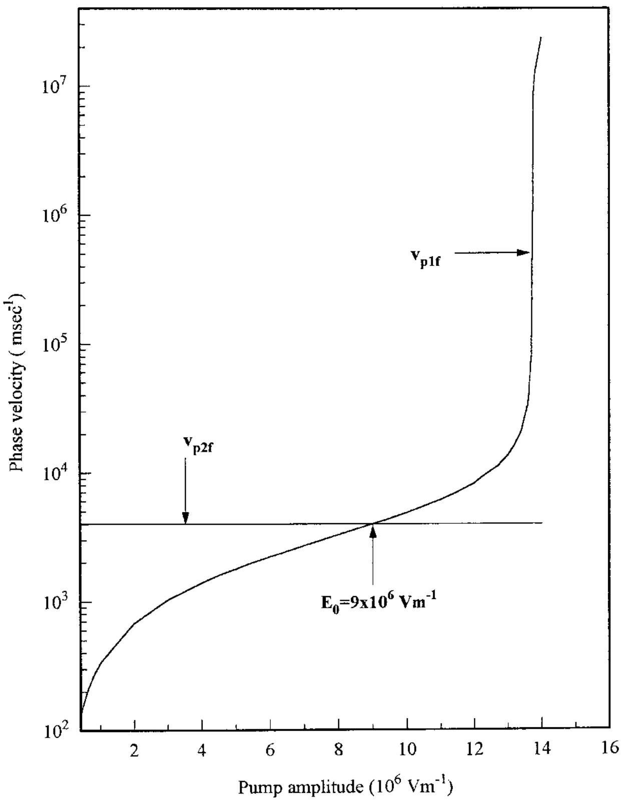

Fig. 3. Variation of phase velocities (v_{p1f}, v_{p2f}) with pump amplitude E_0 when $\omega_c = 0.5\omega_0$ and $n_0 = 10^{22}$ m^{-3}.

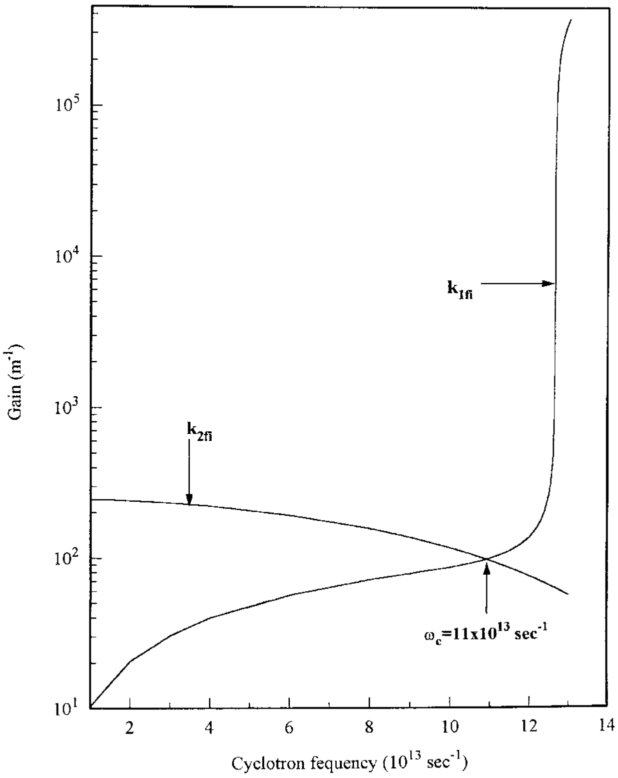

Fig. 4. Variation of gains ($|k_{1\mathrm{fi}}|$, $|k_{2\mathrm{fi}}|$) with cyclotron frequency ω_c when $E_0 = 6 \times 10^6$ V m^{-1} and $n_0 = 10^{22}$ m^{-3}.

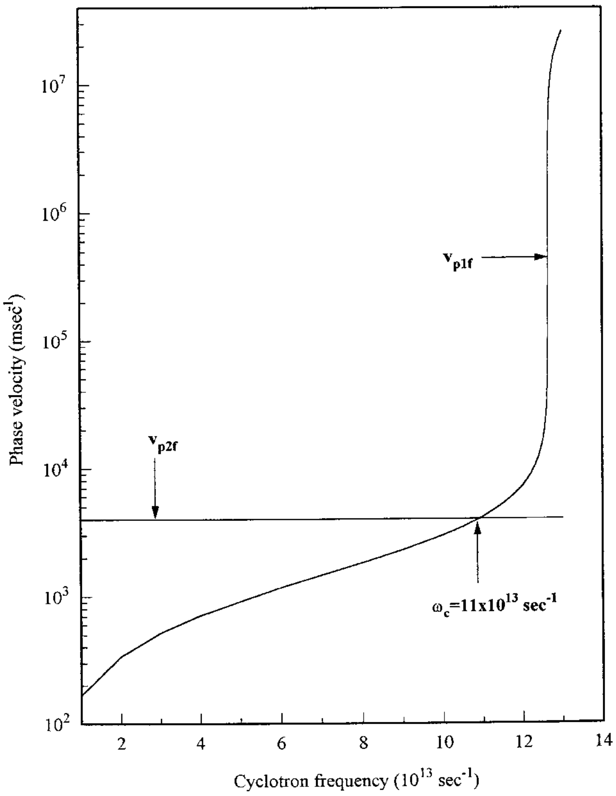

Fig. 5. Variation of phase velocities (v_{p1f}, v_{p2f}) with cyclotron frequency ω_c when $E_0 = 6 \times 10^6$ V m^{-1} and $n_0 = 10^{22}$ m^{-3}.

come into picture and warrants modification in present non-relativistic analysis, accordingly. Apart from this, the approximation $T_e \approx T_0$ in the analysis also forbids E_0 to attain higher values.

In Figures 4 and 5, we have plotted the gains ($|k_{1\mathrm{fi}}|$, $|k_{2\mathrm{fi}}|$) and phase velocities (v_{p1f}, v_{p2f}) of the forward acoustic modes against the variations of static magnetic field $\mathbf{B}_\mathrm{s}$ (*via* ω_c) at constant pump amplitude E_0 ($= 6 \times 10^6$ V m^{-1}). The gain $|k_{1\mathrm{fi}}|$ increases with decreasing slope while the other gain $|k_{2\mathrm{fi}}|$ decreases with increasing slope until they meet at $\omega_\mathrm{c} \approx 11 \times 10^{13}$ s^{-1}. Obviously at this value of ω_c, E_0 ($= 6 \times 10^6$ V m^{-1}) becomes the critical pump amplitude E_cr and thus results in a singularity. Beyond this point, $|k_{1\mathrm{fi}}|$ increases rapidly with increasing slope. The other gain $|k_{2\mathrm{fi}}|$ maintains its decreasing trend. The variations of $|k_{1\mathrm{fi}}|$ and $|k_{2\mathrm{fi}}|$ could be easily understood from the imaginary parts of the equations (27a, 27b). The phase velocities v_{p1f} and v_{p2f} (Eqs. (28a, 28b)) follow almost the same variation patterns as those of their respective gains. Here we again encounter the same situation as in Figures 2 and 3; one may expect arbitrarily large gain $|k_{1\mathrm{fi}}|$ by slightly increasing the ω_c beyond the critical point ($E_0 = 6 \times 10^6$ V m^{-1}), but this possibility is again ruled out because at large magnetic field, the cyclotron resonance effects start playing vital role and thus the present analysis starts losing its validity.

In summary, we have analytically investigated the possibility of excitation and convective amplification of AWs as a result of PI of intense pump with the magnetoplasma in PES. This phenomenon in the medium generates two amplifying and two attenuating acoustic modes. The gains of the amplifying modes decrease up to the critical point E_cr beyond which one of the mode found shooting up abruptly while the other maintains the decreasing trend. The presence of strong magnetic field ($\mathbf{B}_\mathrm{s}$) is found to shift the critical point towards lower pump amplitude. However, the occurrence of magnetic resonance effects restricts $\mathbf{B}_\mathrm{s}$ to increase beyond a limit. A plateau of gains, observed over a considerable range of pump amplitude around E_cr, can be potentially exploited for the simultaneous amplification of the acoustic modes. This region may also enable us to fabricate parametric oscillators and amplifiers with stable gains by suitably choosing critical pump amplitude along with the aid of an appropriate transverse magnetic field. The above discussed amplification/attenuation characteristics of the AWs may be attributed to heating of the carriers by the pump and the resultant temperature dependence of MTCF of the electrons.

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